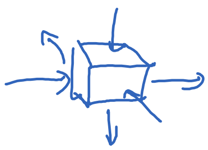
Lecture 35

Monday, April 5, 2021 1:28 PM

* Prayer
* Spiritual thought – General Conference
* Answering questions....

Vector field
- divergence : scalar function
- curl : vector field

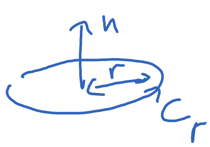


$$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$= \underbrace{\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle}_{\nabla} \cdot \underbrace{\langle P, Q, R \rangle}_{F}$$

Curl : the direction around which F rotates the most.

$$(\operatorname{curl} F) \cdot n = \lim_{r \to 0} \frac{1}{A_r} \int_{C_r} F \cdot dr$$



Ex: $F = \langle 2x-y, 2y+z, z^2+x \rangle$

$\operatorname{div} F = 2+2+2z = 4+2z$

$\operatorname{curl} F = \langle 0-1, 0-1, 0-0 \rangle = \langle -1, -1, 0 \rangle$

Note: rotation in macroscopic scale $\neq$ rotation in microscopic scale

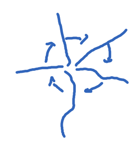
$F(x,y) = \langle y, 0, 0 \rangle$ doesn't seem to rotate in macroscopic scale, but not microscopic scale.

$F(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0 \right\rangle$ doesn't seem to rotate in microscopic scale, but not in macroscopic scale.

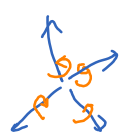
__Properties__:

$\text{curl}(\text{grad}) = 0$

$\text{div}(\text{curl}) = 0$



level sets can't intersect each other.



curl

a vector field (around a point) can't rotate around more than one axis.

$\text{curl} = 0 \rightsquigarrow$ not rotating

__Ex__: Is $G = \langle x, y, z \rangle$ a conservative vector field?

__Curl in 2D__

$F = \langle P, Q \rangle \rightsquigarrow \langle P, Q, 0 \rangle$

$\text{curl}\, F = \langle 0, 0, Q_x - P_y \rangle = (Q_x - P_y)\vec{k}$.

$$\iint \underbrace{\text{curl} F}_{(Q_x - P_y)\vec{k}} \cdot \vec{k}\, dA = \iint (Q_x - P_y)\, dA$$

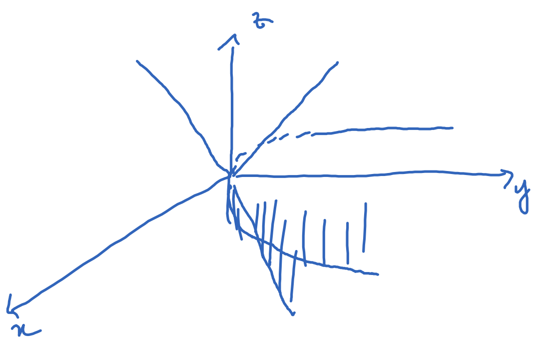
$d\tilde{r} = \langle dy, -dx \rangle$

$$\int_C F \cdot d\tilde{r} = \int_C P\,dy - Q\,dx$$

$$= \iint_D (P_x + Q_y)\,dA = \underbrace{\iint_D \operatorname{div} F\,dA}_{\text{Gauss thm}}$$

Surface $\rightsquigarrow$ not just a graph of a function

Ex

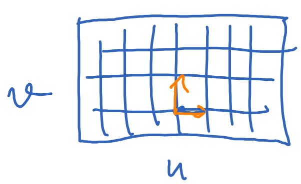


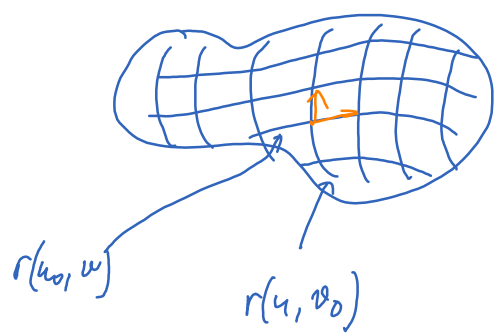
Mobius strip:

$$\begin{cases} x = 2\cos\theta + r\cos\frac{\theta}{2} \\ y = 2\sin\theta + r\cos\frac{\theta}{2} \\ z = r\sin\frac{\theta}{2} \end{cases}$$

$$-\frac{1}{2} \le r \le \frac{1}{2}, \quad 0 \le \theta \le 2\pi$$

$r = r(u,v)$

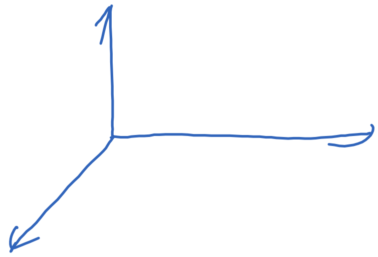




change of r when $v = v_0$ and u changes:

$$\frac{\partial r}{\partial u}\,du$$

Ex:



plane $x+y+z=1$

inside $x^2+y^2=3$

Torus: